CEE 123 Transport Systems 3: Planning & Forecasting

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Homework #1. Review of Pre-requisite Material [S O L U T I O N S]

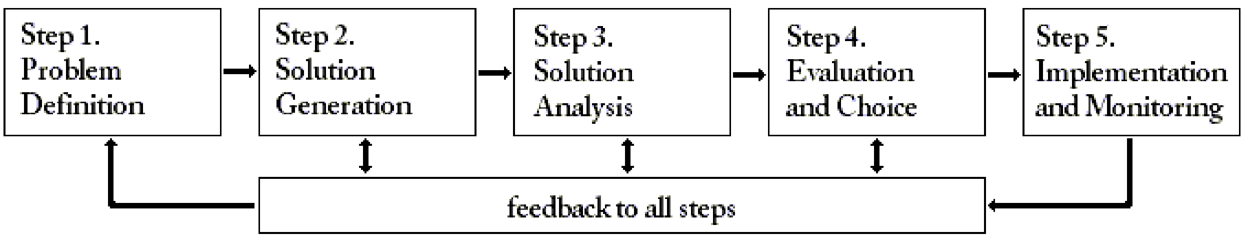
Problem 1. [CEE121] Travel Forecasting [10 points]

Review Mannering *et al*. (2004) Chapter 8. Read 8.1-8.3; skim 8.4-8.5; read 8.6; skim 8.7 and Appendix 8A (note: this book was used in CEE121). The same material is available in most transportation texts and on-line (see, for example, The Four Step Model (MGMcNally) or Travel Forecasting Primer (Bierborn)).

Answer the following questions ***in your own words***:

a. What is the Transportation Planning Process?

TPP is the standard problem solving approach for identifying and resolving transportation problems at all spatial layers.



The Transportation Planning Process

b. What are the **steps** in the sequential approach to forecasting future travel?

Mannering et al. identify the Four Step Model as (1) trip generation, (2) destination choice (trip distribution), (3) mode choice and (4) route choice (trip assignment).

c. What are the **inputs** and **outputs** of each forecasting step?

The inputs to trip generation are demographic and socio-economic variables describing the activity system; the outputs of trip generation are the frequencies of trip origins and destinations by zone, typically categorized by activity type. The inputs to destination choice (trip distribution) are the generation outputs (trip origins and destinations) and travel times between zones; the trip distribution outputs are the destinations for the total generated trips of an origin zone. Mode choice factors the resulting trip tables by mode shares. The input to route choice (trip assignment) are (mode-specific) trip tables from trip distribution (and mode choice) as well as the transport network (paths and path travel times); the output is the set of (equilibrated) flows on the network (link volumes and travel times).

d. What is a **link performance function**? What role does it play in travel forecasting?

The fundamental speed/density/flow relationship for a facility translates into a non-linear link performance function, with travel time increasing at an increasing rate as link volume approaches link capacity. LPFs represent the performance of a link relative to the demand for travel. The equilibration of demand and performance produces the flows (volumes and travel times) on a network.

e. What is the difference between **User Equilibrium** and **System Optimal** route choice formulations?

UE (a result of Wardrop's 1st principle) states that routes utilized by travelers for a given O-D pair have equal travel times (a Nash equilibrium). SO simply states that total system travel time is a minimum that, in general, is not an equilibrium state.

Problem 2. [CEE11, 121] Statistical Methods [20 points]
The following speed and density data was collected on a local freeway segment.

```
   Table 2. Speed and Density Measurements (2025)
----------------------------------------------------------------
 Observation   Units      1    2    3    4    5    6    7    8    9   10
----------------------------------------------------------------
 Speed   SMS   mph       50   45   40   30   25   50   35   35   25   20
 Density  D    veh/mi    10   20   35   40   70   15   40   50   80  100
----------------------------------------------------------------
```

a. **Estimate** a linear speed-density regression model with X = density (D) and Y = Speed (u_s). You may perform the calculations by hand or use available software (include model input and output).

Sample hand calculations (any software can be used):

```
----------------------------------------------
 Obs       X       Y    X-sq    Y-sq      XY
         Den     SMS
----------------------------------------------
   1      10      50     100    2500     500
   2      20      45     400    2025     900
   3      35      40    1225    1600    1400
   4      40      30    1600     900    1200
   5      70      25    4900     625    1750
   6      15      50     225    2500     750
   7      40      35    1600    1225    1400
   8      50      35    2500    1225    1750
   9      80      25    6400     625    2000
  10     100      20   10000     400    2000
----------------------------------------------
 Sum     460     355   28950   13625   13650
 Mean  X'=46   Y'=35.5
 S.D.     29.4    10.7
----------------------------------------------

b1 = [Sum{XY} - nX'Y']/[Sum{X}sq - nX'sq]
   = [13650-(10)(46)(35.5)]/[28950-(10)(2116)]
   = -0.3440

b0 = Y' - b1 X' = 35.5 - (-0.3440)(46)
   = 51.3240

R = [Sum{XY} - nX'Y']/[Sqrt(Sum{Xsq}-nX'sq) Sqrt(Sum{Ysq}-nY'sq)]

             [13650-(10)(46)(35.5)]
  = --------------------------------------------------
    [Sqrt(28950-(10)(2116)) Sqrt(13625-(10)(1260.25))]

  = -0.9496

R-sq = R(R)
     = 0.9017

Sest = Sqrt [ (Sum{Ysq} - b0(Sum{Y}) - b1(Sum{XY}) ]/(n-k-1) ]
     = Sqrt [{13625-(51.324)(355)-(-0.3440)(13650)}/8]
     = Sqrt [100.58/8]
     = 3.5458

Sb = Sest / [Sx Sqrt(n-1)]
   = 3.5458 / [(29.4)(3)]
   = 0.0402

t = b1/Sb
  = -0.3440/0.0402
  = -8.5572
```

```
Model:
Y = 51.3240 - 0.3440 X
```

b. **Define** and **find** mean free speed (u_f) and jam density (D_j) and express the results in Greenshield's format:

$u_s = u_f (1 - D / D_j)$

Mean Free Speed *is the average speed of vehicles traveling unimpeded on a defined section of roadway.* ***Jam Density*** *is a facility's maximum density (vehicles per mile), where spacing and space mean speed approach zero (cars are "bumper to bumper").*

```
Greenshield's:  us  =  uf ( 1 - D/Dj )
                    =  51.3 - 0.3440 D =  51.3 [ 1 - 0.0067 D]
                    =  51.3 [ 1 - (D/150) ]
thus:           uf  =  51.3 mph
and             Dj  =  150 vpm
```

c. Is the model **significant**? What **specific tests** support your contention?

Below is the Excel output for the regression above:

```
b1   =  -0.344030809    b0  =  51.3254172
se1  =   0.040157231    se0 =   2.160668026
R2   =   0.901714002    SEE =   3.544316447
F    =  73.39511416     df  =   8
ssr  = 922.0025674      sse = 100.4974326
```

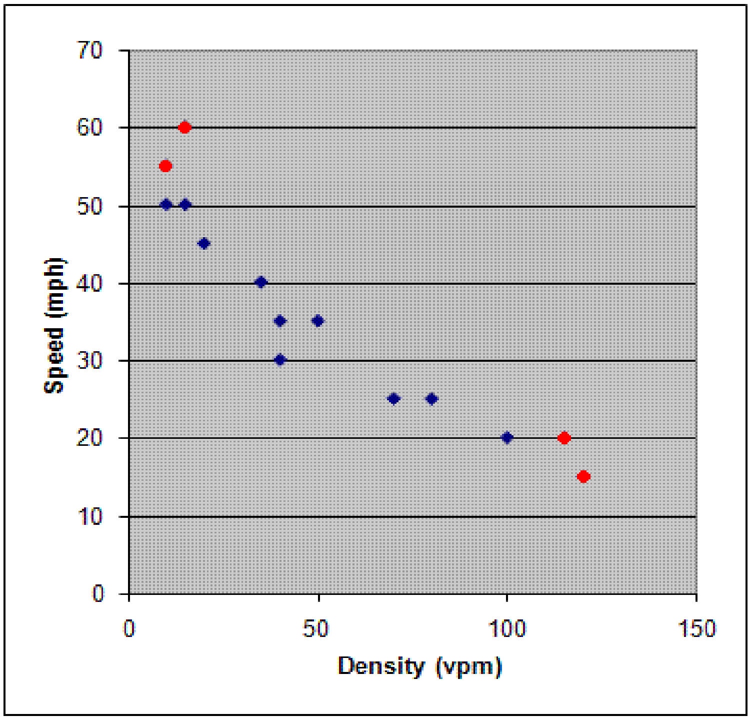
From the coefficients and associated standard errors, the t scores can be computed, showing that both the constant (t=23.75) and the density coefficient (t=8.56) are significantly (at 5%) as is the model's F-stat (73.40). The model is significant.

d. Consider four additional data points: {S,D} = {60,15},{15,120},{20,115},{55,10}. **How** will these points affect the estimated model? Does a **plot** suggest that the linear Greenshield's model might not be appropriate?

Below is the Excel output for the new regression:

```
b1   =   -0.3403    b0  =  53.5730
se1  =    0.0380    se0 =   2.4247
R2   =    0.8699    SEE =   5.3701
F    =   80.2032    df  =  12
ssr  = 2312.8766    see = 346.0524
```

Based on the plot, the speed density curve is not strictly linear as above, thus, a linear regfression may not be appropriate.



Problem 3. [CEE121] Performance-Demand Equilibration [10 points]
Two single-link paths connect an origin and destination with performance functions:

$t_1 = 1 + 0.5\ x_1$
$t_2 = 2 + 1.0\ x_2$

with time t in minutes (min.) and volume x in thousands of vehicles per hour (kvph).

a. Determine UE flows if the total origin-to-destination demand is 800 veh/hr
b. Determine UE flows if the total origin-to-destination demand is 3,000 veh/hr
c. Calculate the total vehicle-hours of travel for both case (a) and (b)
d. Referring to Problem 1, how does this problem fit the sequential forecasting process? What elements are demand and what elements are supply?

Solutions:

(a) Determine UE flows if the total origin-to-destination demand is 0.8 kvph.

Test if both paths are used at the specified total flow under UE assumptions:

1. At T=0, t_1=1 and t_2=2 , thus Path #1 is used first.
2. At t_1=2, x_1=2.0 kvph. Until this volume is met (when $t_1 = t_2$), all traffic uses Path #1.
3. At T=0.8, only Path 1 is used so t_1=1.4 min, x_1=0.8 kvph.

(b) Determine UE flows if the total origin-to-destination demand is 3.0 kvph.

From part (a), both paths are used when T is greater than 2.0 kvph. Solve for T=3.0 kvph.

$t_1 = 1 + 0.5\ x_1$
$t_2 = 2 + 1.0\ x_2$
$t_1 = t_2$
$x_1 + x_2 = 3.0$

$1 + 0.5\ x_1 = 2 + (3.0 - x_1)$
$x_1 = 2.67$ kvph
$t_1 = 1 + 0.5\ (2.67) = 2.33$ min
$x_2 = 0.33$ kvph
$t_2 = 2.33$ min

(c) Calculate the total vehicle-hours of travel for both case (a) and (b)

Total Vehicle-Hours Traveled (TVHT) = $[x_1(t_1) + x_2(t_2)]/60$
Case (a): TVHT(a) = 800(1.4)/60 = 18.67 vht
Case (b): TVHT(b) = [2667(2.33) + 333(2.33)]/60 = 3000(2.33) = 116.67 vht

NOTE: You will need to perform similar calculations throughout the quarter.

(d) Referring to Problem 1, how does this problem fit the sequential forecasting process?
What elements are demand and what elements are supply?

Total OD demand is fixed, at 800 vph for case (a) and at 3000 vph for case (b), which corresponds to the firsts three steps of the Four Step Model (FSM). This analysis thus corresponds to Step 4 (route choice, or trip assignment) which, as for the basic FSM, is an equilibration of route choice only. The link performance functions provide the supply side expresions.

Problem 4. [CEE110] Project Evaluation [10 points]
In the final task of the CEE123 term project, teams will compare future alternative transportation systems in terms of system performance and system cost relative to a "No Build" alternative. There are several project evaluation techniques that can be utilized.

In 2000, Miasma Beach considered three different intersection improvement alternatives for the intersection of 1st Street and Mountain Boulevard. The City defined the following goals for this project: (1) improve travel speeds; (2) increase safety; and (3) reduce operating expenses for motorists. Project benefits and costs have been quantified and summarized. Each alternative has a design life of 50 years and the discount rate is 3 percent.

Table 1. 1st & Mountain Intersection Improvement Project

Alt	Project Costs		Annual Project Benefits		
	Initial Construction	Annual M&O	Savings in Travel Time	Accident Savings	Savings in Operations
1	$185,000	$1,500	$3,000	$5,000	$500
2	220,000	2,500	6,500	5,000	500
3	310,000	3,000	6,000	7,000	2,800

Note: except for initial construction, all benefits, as well as Maintenance & Operations costs are annual.

Which alternative should be selected? Apply **two** project evaluation methods, choosing from: (a) Net Present Worth; (b) Benefit Cost Analysis; or (c) Rate of Return.

1.1 Solution: Net Present Worth

To compute present worth of fixed annual payment: $(P/A, r, n) = [\ (1+r)^n - 1\ ] / [\ r(1+r)^n]$

$(P/A, 0.03, 50) = [\ (1+0.03)^{50} - 1\ ] / [\ 0.03(1+0.03)^{50}] = 25.729$

A1:
NPW(A1) = [-185,000 - (1500)(P/A, 0.03, 50)] + [(3000 + 5000 + 500)(P/A, 0.03, 50)]

NPW(A1) = [-185,000 - (1500)(25.729)] + [(3000 + 5000 + 500)(25.729)]
NPW(A1) = [-185,000 - 38,594] + [218,697]
NPW(A1) = -223,594 + 218,697 = -$4,897

A2:
NPW(A2) = [-220,000 + (-2500)(P/A, 0.03, 50)] + [6500 + 5000 + 500)(P/A, 0.03, 50)]
NPW(A2) = [-220,000 + (-2500)(25.729)] + [6500 + 5000 + 500)(25.729)]
NPW(A2) = [-220,000 - 64,323] + [308,748] NPW(A2) = -284,323 + 308,748 = $24,426

A3:
NPW(A3) = [-310,000 - (3000)(P/A, 0.03, 50)] + [6000 + 7000 + 2800)(P/A, 0.03, 50)]
NPW(A3) = [-310,000 - (3000)(25.729)] + [6000 + 7000 + 2800)(25.729)]
NPW(A3) = [-310,000 - 77,187] + [406,518] NPW(A3) = -387,187 + 406,518 = $19,331

The project with the greatest Net Present Worth is **A2**.

1.2 Solution: Benefit Cost Analysis

With multiple alternatives, use Incremental Benefit Cost Analysis, with alternatives examined in order of increasing cost (here, A1, then A2, and then A3). Note that the Present worth of all annual payments has already been computed as part of the solution via the Net Present Worth approach.

• Compare **A1** with **A0**, the No Build alternative:

IBCR(A1 vs A0) = [218,697 - 0] / [223,594 - 0] = 0.98

Since (IBCR < 1.0), reject A1 in favor of **A0**.

• Compare **A2** with **A0**:

IBCR(A2 vs A0) = [308,748 - 0] / [284,323 - 0] = 1.09

Since (IBCR > 1.0), select **A2** over A0.

• Compare **A3** with **A2**:

IBCR(A3 vs A2) = [406,518 - 308,748] / [387,187 - 284,323] = 0.95

Since (IBCR < 1.0), reject A3 in favor of **A2**

The preferred project based on Benefit Cost Analysis is **A2**.

1.3 Solution: Rate of Return Note: Calculator

With ROR, find the interest rate for which the Net Present Worth is zero. Here, as for Benefit Cost Analysis, all alternatives are compared in terms of increasing cost.

• NPW(A1 vs A0) = 0 = -185,000 + (-1500 + 3000 + 5000 + 500) (P/A, r, 50)

RoR: (P/A, r, 50) = 185,000 / 7,000 = 26.429, thus, r = 2.8 percent

RoR is less than current discount rate of 3% so do not make the investment in A1. Keep A0.

• NPW(A2 vs A0) = 0 = -220,000 + (-2500 + 6500 + 5000 + 500) (P/A, r, 50)

RoR: (P/A, r, 50) = 220,000 / 9,500 = 23.158, thus, r = 3.65 percent

RoR is greater than current discount rate of 3% so Alternative A1 is selected over A0.

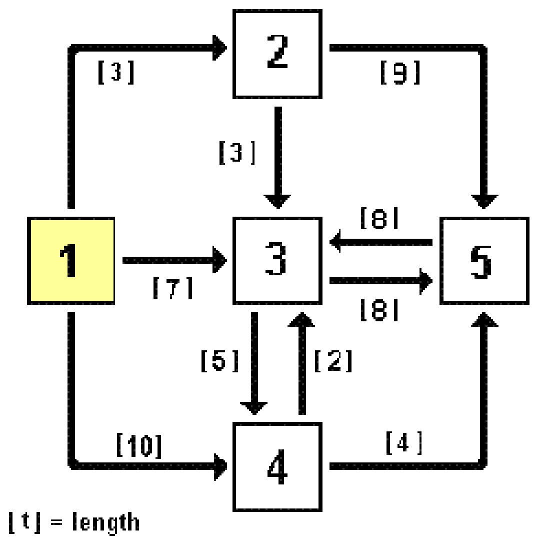
• NPW(A3 vs A2) = 0 = -(310,000 - 220,000) + (21,800 - 9,500) (P/A, r, 50)

RoR: (P/A, r, 50) = 90,000 / 3,300 = 27.273, thus, r = 2.7 percent

RoR is less than the current discount rate of 3% so reject A3 in favor of **A2**

All three project evaluation techniques produce the same result.

Problem 5. [CEE111] Network Models and Optimization [10 points]
Formulate the linear program to find the minimum path from node 1 to node 5 (no need to solve).



Math Programming Formulation (inbound flow is positive)

Min C = $\Sigma_{ij}\, x_{ij}\, c_{ij}$
subject to:
$\Sigma_i\, x_{is} - \Sigma_j\, x_{sj} \geq -1$ . . . for each origin node s
$\Sigma_i\, x_{ik} - \Sigma_j\, x_{kj} = 0$ . . . for each intermediate node k
$\Sigma_i\, x_{it} - \Sigma_j\, x_{tj} \geq +1$ . . . for each destination node t

Here, c_{ij} = the cost on link (i,j) and x_{ij} is the flow on link (i,j). At the optimum solution, $x_{ij} = 1$ for links on the minimum path (and 0 otherwise). The value of the objective function is the length of the minimum path.

Dual Formulation

Max D = $w_t - w_s$
subject to:
$w_j - w_i \leq C_{ij}$ . . . for all links (i,j)

Here, w_j is the node label containing the cumulative travel time to node j and c_{ij} is the cost on link (i,j). The two formulations are equivalent. The optimum value of D equals that of C, the length of the minimum path.

Math Programming Formulation (inbound flow is positive)

```
Min C =
 3x12 + 7x13 +10x14 + 3x23 + 9x25 + 5x34 + 8x35 + 2x43 + 4x45 + 8x53
Subj to:
 -x12 -  x13 -  x14                                              >=-1
  x12              -  x23 -  x25                                  = 0
         x13          +x23        -  x34 -  x35  + x43        + x53   = 0
                x14               +  x34        - x43 -  x45          = 0
                             x25        +  x35        +  x45  - x53  >= 1
```

Last Updated: 1 April 2025